Color-Swap Models for Non-growing Scale-free Networks

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Introduction

Non-growing scale-free networks represent an interesting class of complex networks. Compared to growing networks, which have been studied in many different scenarios with great success [1, 2, 3], they aim at modeling situations where a complex topology different from purely random is emerging but no growth process is present. There are many examples, mainly in biology (see [4] and the references therein), where instead of growth one can observe a self-organizing stochastic process which moves some network connections to different nodes, and the number of nodes and edges stays statistically bounded within some interval.

Although various aspects of the dynamic behavior of non-growing (equilibrium) networks were well studied [5] and several stochastic process candidates have been proposed, they provide the scale-free property only under special conditions. The discrepancies between real networks and most model designs can be summarized as follows: (i) Multigraph: the processes in [5, 4] allow multiple edges and/or self-loops between the nodes, whereas in most real networks this phenomena does not exist. (ii) Connection (edge) growth: the process defined in [6] will reach a scale-free state only in certain range of the node degree distribution and also the number of edges grows considerably before it stabilizes. (iii) Condensation: the networks go through a scale-free state but later condensate to a state where a small fraction of nodes posses all the edges [5]. (iv) Structural constrains: the stochastic process defined in [4] reveals very interesting aspects, however the scale-free character is a result of structural constrains and special initial conditions.

We present an edge-colored graph concept aimed at developing processes, generating non-growing scale-free networks, which would lack the disadvantages of the processes known so far; in particular, we propose a processes "Color-Swap" which avoids the above mentioned problems (i), (ii), (iv) and improves on the problem (iii). A more detailed discussion and extensive experimental results we made available in [7].

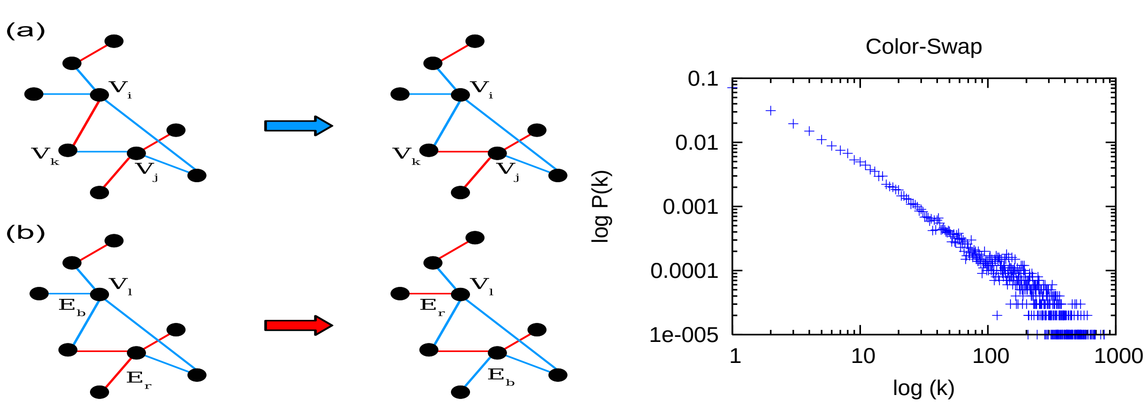

Fig. 1: (Color online) Color-Swap process: (a) Blue rewiring phase, (b) red rewiring phase; for detailed process description see the text. (c) Right: Equilibrium blue degree distribution obtained by simulation of the Color-Swap process. Parameters used: $N = 100000, L_{blue} = L_{red} = 300000$, thus $L_{blue} \ll L_{black}$.

Color-Swap Models

To model non-growing complex networks we use an edge-colored complete graph with $\binom{N}{2}$ edges. The edges are colored in black, blue and red, where black edges represent non-edges (no connection) in the modeled network, blue edges represent the real connections and red edges represent potential connections (see conclusion for interpretation). The edge sets and the number of edges is denoted as $|E_{<color>}| = L_{<color>}$, consequently networks with low average degree have $L_{blue} \ll L_{black}$. We define also the blue and red degrees of a node as the number of incident edges having the respective color. The only operation allowed is color exchange (swap) between two edges. We study a process called Color-Swap defined as follows (see also Fig. 1):

1. Initialization: Start with a simple graph $G(V, E_{blue} \cup E_{red} \cup E_{black}) = K^{|V|}$, repeat the steps 2, 3.
2. Blue Rewiring Phase: Uniformly at random select two nodes V_i and $V_j \neq V_i$. For each blue neighbor V_k of V_j swap the edge (color exchange) V_j-V_k with the edge V_k-V_i (see a) in Figure 1).
3. Red Rewiring Phase: Select a vertex V_l preferentially with a probability proportional to $f(k_{blue}) = k_{blue}/2L_{blue}$ where k_{blue} is the blue degree of the vertex V_l. Next, uniformly at random select a blue edge E_b incident on it and swap it with an uniformly at random chosen red edge E_r (see b) in Fig. 1).

Because every edge has an unique color and the allowed operations do not change this feature, a creation of multiple edges and self-loops is not possible. Simulations show that the Color-Swap process generates a scale-free network independently of initial conditions and the network size (see Fig. 1).

Conclusion

Table 1 summarizes the experimental results on various processes we simulated. The experiments show that the Color-Swap process is a quickly converging process, and compared to other known processes it improves on the above described problems. In particular, due to the mechanism of color exchange, it stays naturally in the class of simple graphs and it controls the growth of hubs leading to a scale-free structure. The concept of colored edges might be useful in modeling networks with various connection types. For example one can imagine to represent the real connections of a technological network (such as the Internet) by blue edges, whereas the back-up links are modeled by the potential/red edges. The color models can also be generalized for more than three colors and it is imaginable to use them in order to model different types of protein interactions in biological networks. Therefore the Color-Swap model and the concept of colors in general are interesting candidates for further study in the domain of stochastic processes generating non-growing scale-free networks.

References

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	SESP [5] [7]	**SGESP** [7]	**VADE** [7]	**Merg.-Reg.** [6]	**Park et al.** [4]	**Color-Swap**
Condensation	Yes	Yes	No	No	No	No
Structural constraints	No	No	No	No	Yes	No
Edges	Constant	Constant	Bounded	Too many	Constant	Constant
Simple graph	No	Yes	Yes	Yes	No	Yes
Convergence	Slow	Slow	Slow	Fast	Fast	Fast

Tab. 1: Comparison of models for non-growing complex networks. SGESP is the same process as SESP except that the rewiring step does not occur if a multiple edge would be created. VADE is a new process described in [7] and the references therein.